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LETTER TO THE EDITOR

Inverse problems in classical scattering theory for centrosymmetric electric and magnetic fields

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Abstract. New results are obtained in general scattering theory. The inverse scattering problem is solved in quadratures at a fixed impact parameter in classical mechanics. The inverse problem is analysed for a magnetic field at a fixed angular momentum and the magnetoelectric analogy is formulated. Focusing force electric and magnetic fields are constructed for charged particles with a specified angular momentum. The generalised optical–mechanical analogy is established. This analogy allows us to investigate some previously considered problems in the relativistic case.

For the classical scattering by a centrosymmetric field the deflection angle χ depends only on the impact parameter b (or the angular momentum l) and the total energy E (or the infinity momentum $k=(2mE)^{1/2}$). For an electric field the potential $V(r)$ can be reconstructed if one knows

(1) $\chi(E)$ at fixed l (the Hoyt (1939) algorithm);

(2) $\chi(b)$ at fixed E (the Firsov (1953) algorithm).

The third reconstruction algorithm is proposed here when $\chi(k)$ is known at fixed b, namely

$$r(\varkappa)=b\sec\frac{1}{\pi}\int_0^{\varkappa}\mathrm{d}k(\varkappa^2-k^2)^{-1/2}\chi(k),$$

$$\varkappa\equiv r(r^2-b^2)^{-1/2}(2mV)^{1/2}$$

which specifies $V(r)$ in an implicit form. All these algorithms are based on Abel-type integral transformations.

Using the Hoyt algorithm for the fixed l case one can construct the following (in the same manner as for the fixed E case in Demkov *et al* (1971)).

(a) The analogue of the Luneburg (1964) lens with radius R, potential

$$V=(l^2/8mR^2)(3-4R/r+r^2/(2R-r)^2),\qquad r<R,\qquad V=0,\qquad r>R;$$

the latter force field focuses particles of mass m with given l onto the edge of the lens.

(b) The analogue of the Maxwell 'fish eye'—the cut-off Coulomb potential

$$V=(l^2/mR^2)(1-R/r),\qquad r<R,\qquad V=0,\qquad r>R;$$

in this focusing system the source and the focus are at diametrically opposite points.

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(c) The lens of the 'cat's-eye' type (the back-scattering potential)

$$V=-(l^2/4mR^2)(1-\tfrac{3}{2}R^2/r^2+\tfrac{1}{2}r^2/R^2), \qquad r<R, \qquad V=0, \qquad r>R$$

for which $\chi=\pi$ for all $b<R$.

For the classical scattering by a magnetic field we consider the axially symmetric case when in the cylindrical coordinate system ρ, φ, Z the vector potential components $A_\rho=A_Z=0$, $A_\varphi\equiv A(\rho)$ depends only on ρ. A magnetic field $H=\cot A$ is directed along the Z axis, while all trajectories of the incident particles of charge e are assumed to lie in the (x, y) plane. The inversion formula for this problem could be obtained only for the fixed angular momentum:

$$g(\rho)=\exp\frac{1}{\pi}\int_{\rho g}^{\infty}\mathrm{d}b(b^2-\rho^2g^2)^{-1/2}\chi(b),$$

$$g\equiv l/(l-e\rho A)$$

which coincides by its form with Firsov's inversion formula. This coincidence allows one to formulate the magnetoelectric analogy: the trajectories of particles of charge e and total energy E in an axially symmetric electric scattering field with the scalar potential $\phi(\rho)$ coincide with the trajectories of particles of the same charge and angular momentum l in an axially symmetric magnetic scattering field with the vector potential

$$A_\rho=A_Z=0, \qquad A_\varphi\equiv A(\rho)=l(e\rho)^{-1}[1-(1-e\phi/E)^{-1/2}].$$

Thus we have the magnetic analogue of the Luneburg lens

$$A=l(e\rho)^{-1}[1-(2-\rho^2/R^2)^{-1/2}, \qquad \rho<R, \qquad A=0, \qquad \rho>R;$$

that of the Maxwell 'fish-eye'

$$A=l(2e\rho)^{-1}(1-\rho^2/R^2), \qquad \rho<R, \qquad A=0, \qquad \rho>R$$

and the back-scattering magnetic potential ('cat's-eye')

$$A=l(e\rho)^{-}1[1-(2R/\rho-1)^{-1/2}], \qquad \rho<R, \qquad A=0, \qquad \rho>R.$$

Firsov's inversion formula can be generalised easily to the relativistic case when the particle velocity is comparable with the light velocity. Both non-relativistic and relativistic movement problems in classical mechanics can be considered as optical problems after the introduction of the refraction index n proportional to the particle momentum at each point. This statement is correct for arbitrary non-central potential fields $V(r)$ because it is a consequence of the Maupertuis variational principle. In the relativisitic case

$$n=(E^2-m^2)^{-1/2}((E-V)^2-m^2)^{1/2}.$$

This formula expresses the generalised optical–mechanical analogy.

One can easily construct the relativistic analogue of the Luneburg lens, the Maxwell 'fish-eye' and the 'cat's-eye' and also the relativistic generalisation of the wave 'fish-eye' problem for a particle with total energy $E=m$. The wave generalisation of the 'fish-eye' problem in the non-relativistic case has been found by Demkov and Ostrovskii (1971).

All the variants of the inversion formulae considered here can be used in atomic collisions, in electronic optics and in other regions of physics where the semiclassical conditions are valid (Bogdanov and Demkov 1982a, b). These inversion algorithms are also interesting for an investigation of the transition from the quantum inverse problem in scattering theory to the classical one.

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